ST. JOSEPH’S COLLEGE FOR WOMEN (AUTONOMOUS), VISAKHAPATNAM

III SEMESTER  **MATHEMATICS** TIME:3HRS/WEEK

M-Ma1-3301(3)  **GROUP THEORY**  MARKS:100

w.e.f AK 2023-2024 (Admitted batch) **SYLLABUS**

**COURSE OBJECTIVES:** To enable the students to –

**CO1:** Recall the definition of a group and its basic properties, Explain the difference between a

Semigroup, a monoid, and a group, interpret composition tables to understand group

operations (**K1 & K2)**

**CO2:** Verify whether a given subset satisfies the subgroup criteria, Determine whether the intersection or union of two subgroups is also a subgroup, investigate the properties of cosets and their significance, Explore the implications of Lagrange’s Theorem on subgroup sizes. **(K3 & K4)**

**CO3:** Formulate the definition of a normal subgroup, Differentiate between proper and improper

normal subgroups, Assess the impact of normal subgroups on group structure, Analyse the

intersection of two normal subgroups. **(K5 & K6)**

**CO4:** Construct examples of homomorphisms, investigate the properties of images and kernels.

Evaluate the impact of isomorphisms and automorphisms on group structures, understand the significance of the fundamental theorem on homomorphisms. **(K3 & K4)**

**CO5:** Recall the definition of permutations and their basic properties, Identify cyclic permutations.

Explain the significance of even and odd permutations, Understand the role of Cayley’s

theorem in group theory. **(K1 & K2)**

**Course Outcomes:** After successful completion of this course, the student will be able to

**CO1:** Define a group and explain its fundamental properties, Understand the concept of order

within a group, Describe the algebraic structure of a semigroup and a monoid, Explain the

composition tables for groups **(K1 & K2)**

**CO2:** Determine whether a given subset is a subgroup of a group, Apply the criterion for the product of two subgroups to be a subgroup, Analyse the properties of cosets and their relationship to subgroups, calculate the index of a subgroup using Lagrange’s Theorem.

**(K3 & K4)**

**CO3:** Define a normal subgroup and distinguish between proper and improper normal subgroups,

Apply the criterion for a subgroup to be normal, Evaluate the intersection of two normal

subgroups, Understand the concept of normal subgroups in group theory. **(K5 & K6)**

**CO4:** Define a homomorphism between groups, Understand the image and kernel of a

Homomorphism, Explore the properties of isomorphisms and automorphisms, Apply the

fundamental theorem on homomorphisms. **(K3 & K4)**

**CO5:** Define permutations and understand their multiplication, Explain the concept of cyclic

permutations, Differentiate between even and odd permutations, State Cayley’s theorem.

**(K1 & K2)**

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**Course Content**:

**UNIT – I: Groups:** Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

**UNIT – II: Sub Groups:** Complex Definition – Multiplication of two complexes - Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroups-Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – Properties of Cosets – Index of a subgroups of a finite groups – Lagrange’s Theorem.

**UNIT – III**: **Normal Subgroups:** Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups, Sub group of index 2 is a normal sub group

**UNIT – IV: Homomorphisms:** Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

**UNIT – V: Permutations and Cyclic Groups**: Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley’s theorem. Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

**Activities:**

Seminar/ Quiz/ Assignments/ Applications of Group Theory to Real life Problem /Problem Solving

Sessions.

**Text Book:**

Modern Algebra by A.R. Vasishtha and A.K. Vasishtha, Krishna Prakashan Media Pvt. Ltd., Meerut.

**Reference Books:**

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.

2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut

3. Rings and Linear Algebra by Pundir&Pundir, published by PragathiPrakashan

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