ST. JOSEPH’S COLLEGE FOR WOMEN (AUTONOMOUS), VISAKHAPATNAM

III SEMESTER  **MATHEMATICS** TIME: 3Hrs/Week

M-Ma4-3301 (3)  **SPECIAL FUNCTIONS** Max.Marks:100

w.e.f AK 2023-2024 (Admitted Batch) **SYLLABUS**

**Course Objectives:**  To enable the students to

**CO1:** Understand Euler’s integrals and their connection to beta and gamma functions, Explore elementary properties of gamma functions and their transformations,

Investigate Chebyshev polynomials, including their orthogonal properties and

recurrence relations.

**CO2:** Define Legendre polynomials and their significance, Solve Legendre’s equation

and compute Legendre polynomials of degree ‘n’, Explore the generating function

of Legendre polynomials.

**CO3:** Study Hermite differential equations and their solutions, Compute Hermite

polynomials and understand their generating function, Investigate the orthogonal

properties and recurrence formulas for Hermite polynomials.

**CO4:** Define Bessel’s equation and its solutions, Explore Bessel functions of the first and

second kinds for various orders, Understand the integration of Bessel’s equation in

series and the recurrence formulas for Bessel functions.

**CO5:** Learn about power series and their properties, including the radius of convergence,

Introduce power series solutions for ordinary differential equations, Distinguish

between ordinary and singular points, regular and irregular singular points, and their

impact on power series solutions.

**Course Outcomes:** **Students after successful completion of the course will be able to:**

**CO1:** Understand the Beta and Gamma functions, their properties and relation between

these two functions, understand the orthogonal properties of Chebyshev

polynomials and recurrence relations. **(Unit-I: K2 & K4)**

**CO2:** Solve Legendre equation and write the Legendre equation of first kind, also

Find the generating function for Legendre Polynomials, understand the

orthogonal properties of Legendre Polynomials. **(Unit-II: K3 & K4)**

**CO3:** Solve Hermite equation and write the Hermite Polynomial of order (degree) n, also

Find the generating function for Hermite Polynomials, study the orthogonal properties of Hermite Polynomials and recurrence relations. **(Unit-III: K3 & K4)**

**CO4:** Solve Bessel equation and write the Bessel equation of first kind of order n, also find

the generating function for Bessel function understand the orthogonal properties of

Bessel function. **(Unit-IV: K3 & K4)**

**CO5:** Find power series solutions of ordinary differential equations. **(Unit-V: K3)**

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**Course Syllabus**

**UNIT–1: Beta and Gamma functions, Chebyshev polynomials**

* 1. Euler’s Integrals - Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.
  2. Another form of Beta Function, Relation between Beta and Gamma Functions.
  3. Chebyshev polynomials, Orthogonal properties of Chebyshev polynomials, Recurrence relations, Generating functions for Chebyshev polynomials.

**(Chapter 2.9 – 2.15 and 8.1 – 8.8)**

**UNIT–2: Legendre polynomials**

1. Definition, Solution of Legendre’s equation, Legendre polynomial of degree ‘n’, Generating function of Legendre polynomials.
2. Definition of *Pn**x*and *Qn**x*, General solution of Legendre’s Equation (derivations not required) to show that *Pn**x*is the coefficient of *hn*, in the expansion of (1−2𝑥ℎ+h2)-1/2
3. Orthogonal properties of Legendre’s polynomials, Recurrence formulas for Legendre’s Polynomials, Rodrigue’s formula for Legendre’s Polynomials.

**(Chapter 2.1 – 2.5 and 2.7, 2.8 & 2.12)**

**UNIT–3: Hermite Polynomials:**

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, Generating function for Hermite polynomials.
2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.
3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.**(Chapter 6.1 – 6.8)**

**UNIT–4: Bessel’s Equation:**

1. Definition, Solution of Bessel’s equation, Bessel’s function of the first kind of order ‘n’, Bessel’s function of the second kind of order ‘n’.
2. Integration of Bessel’s equation in series for m=0, Definition of Jn(x), Recurrence formulae for Jn(x).
3. Generating function for Jn(x), Orthogonally of Bessel’s functions. **(Chapter 5.1 – 5.7)**

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**UNIT–5: Power Series and Power Series Solutions of Ordinary Differential Equations:**

1. Introduction, summary of useful results, power series, some important facts about the power series, radius of convergence
2. Introduction of power series solutions of ordinary differential equation
3. Ordinary and singular points, regular and irregular singular points, power series solution.

**(Chapter 7.1 – 7.7 and 8.1 to 8.4)**

1. **Prescribed textbook**
   1. J.N.Sharma and Dr.R.K.Gupta, Special functions, Krishna Prakashan Media(P) Ltd. **(Unit-1 to Unit - 4)**
   2. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S.Chand&Company Pvt. Ltd., Ram Nagar, New Delhi-110055. **(Unit – 5)**

**REFERENCE BOOKS:**

1. Shanti Narayan and Dr.P.K.Mittal ,Integral Calculus, S.Chand& Company Pvt.Ltd.,Ram Nagar, New Delhi-110055.
2. George F.Simmons, Differential equations with Applications and Historical Notes, Tata McGRAW-Hill Edition,1994.
3. Shepley L.Ross, Differential equations, Second Edition, JohnWilly &Sons, NewYork, 1974.
4. Web resources suggested by the teacher and college librarian including reading material.

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