**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**I SEMESTER MATHEMATICS**

**M 101 ALGEBRA–I MAX.MARKS:100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To introduce the basic concepts of group theory and study the structure of groups.

CO 2: To introduce the concepts of conjugacy and G sets and prove Cayley theorem. To introduce explicitly the properties of permutation groups.

CO 3: To determine structure of any abelian groups. To determine structure of finite nonabelian groups through Sylow theorems.

CO 4: To introduce concepts of ring theory. To introduce different types of ideals. To apply Zorn’s lemma on the set of ideals.

CO 5: To introduce prime elements and irreducible elements in a commutative integral domain. To study the domains UFD, PID and ED.

**UNIT I:**

**Groups:** Homomorphisms-Subgroups and cosets.

**Normal Subgroups:** Normal subgroups and Quotient groups-Isomorphism theorems-Automorphisms.

12 Hours

(Sections 4.2, 4.3 of the Chapter 4 and sections 5.1 to 5.3 of the Chapter 5 in the Prescribed Text Book.)

**UNIT II:**

**Normal Subgroups:** Conjugacy and G-Sets

**Permutation Groups:** Cyclic Decomposition- Alternating group -Simplicity of .

12 Hours

(Section 5.4 of chapter 5 and sections 7.1 to 7.3 of the Chapter 7 in the Prescribed Text Book.)

**UNIT III:**

**Structure theorems of groups:** Direct Products- finitely generated abelian groups-Invariants of a finite abelian group Sylow theorems.

12 Hours

(Sections 8.1 to 8.4 of the Chapter 8 in the Prescribed Text Book.)

**UNIT IV:**

**Ideals and Homomorphisms:** Ideals, Homomorphisms, Sum and direct sum of ideals- Maximal and Prime Ideals-Nilpotent and Nil Ideals-Zorn’s Lemma.

12 Hours

(Sections 10.1 to10.6 of the Chapter 10 in the Prescribed Text Book.)

**UNIT V:**

**Unique factorization domains and Euclidean domains:** Unique factorization domains, Principal ideal domains, Euclidean domains, Polynomial rings over UFD.

12 Hours

(Sections 11.1 to 11.4 of the Chapter 11 in the Prescribed Text Book.)

**Prescribed Book:**

Basic Abstract Algebra: P. B. Bhattacharya, S. K. Jain and S. R. Nagapaul, second edition, reprinted in India 1997, 2000, 2001.

**Reference Books:**

1. Topics in Algebra: I. N. Herstein, 2nd Edition, John Wiley & Sons.

2. Algebra: Thomas W. Hungerford, Springer.

3. Algebra: Serge Lang, Revised Third Edition, Springer.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**I SEMESTER MATHEMATICS**

**M 102 REAL ANALYSIS–I MAX.MARKS:100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To describe elementary concepts on metric spaces to get the general idea that is relevant to Euclidean spaces.

CO 2: To study the continuity and its properties of real valued functions in metric spaces.

CO 3: To describe the derivatives of real valued functions defined on intervals or segments, and study its properties.

CO 4: To introduce Riemann-Stieltjes integral as a generalization of Riemann integral and discuss the existence of this integral.

CO 5: To study differentiation of integrals and further the extension of integration to vector valued functions.

**UNIT I:**

**Basic Topology:** Metric spaces, Compact sets, Prefect sets, Connected sets.

12 Hours

Chapter 2, Sections 2.15 to 2.47 of the Prescribed Text Book.

**UNIT II:**

**Continuity:** Limits of functions, Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotone functions, Infinite limits and Limits at Infinity.

12 Hours

Chapter 4 of the Prescribed Text Book.

**UNIT III:**

**Differentiation:** The Derivative of a Real Function, Mean Value Theorems, The Continuity of Derivatives, L’Hospital’s Rule, Derivatives of Higher order, Taylor’s theorem, Differentiation of Vector-valued Functions.

Chapter 5 of the Prescribed Text Book.

**UNIT IV:**

**The Riemann-Stieltjes integral:** Definition and Existence of the Integral, Properties of the integral, Change of variable.

12 Hours

Chapter 6, Sections 6.1 to 6.19, of the Prescribed Text Book.

**UNIT V:**

**The Riemann-Stieltjes integral (continued):** Integration and Differentiation, The Fundamental theorem of Calculus, Integration by parts, Integration of vector-valued functions, Rectifiable curves.

12 Hours

Chapter 6, Sections 6.20 to 6.27, of the Prescribed Text Book.

**Prescribed Book:**

Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3rd Edition,1985. Reference: Mathematical Analysis by Tom M. Apostol, Narosa Publishing House, 2nd Edition, 1985.

**Reference Books:**

Tom M. Apostol, Mathematical Analysis, Narosa Publishing House, 2nd Edition, 1985.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**I SEMESTER MATHEMATICS**

**M 103 TOPOLOGY–I MAX.MARKS:100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To get acquaintance with concepts of sets and functions and their properties which are basic tools to study Mathematics.

CO 2: To introduce metric spaces and some elementary concepts in metric spaces.

CO 3: To study the concept of continuous functions and their properties, Euclidean and Unitary spaces.

CO 4: To understand broader concept of topology and topological spaces, as a generalization of metric spaces and study some basic results in topological spaces.

CO 5: To study the concept of compactness and compact spaces. Some important theorems in compact spaces.

**UNIT I:**

**Sets and Functions:** Sets and Set inclusion – The algebra of sets – Functions – Products of sets – Partitions and equivalence relations – Countable sets – Uncountable sets – Partially ordered sets and lattices. (Chapter I: Sections 1 to 8 of the prescribed text book).

14 Hours

**UNIT II:**

**Metric spaces:** The definition and some examples – Open sets – Closed sets – Convergence, Completeness and Baire’s theorem.

10 Hours

(Chapter 2: Sections 9 to 12 of the prescribed text book).

**UNIT III:**

**Metric spaces (Continued):** Continuous mappings, Spaces of continuous functions – Euclidean and unitary spaces. (Chapter 2: Sections 13 to15 of the prescribed text book)

**Topological spaces:** The definition and some examples – Elementary concepts.

(Chapter 3: Sections 16 to 17 of the prescribed text book). 12 Hours

**UNIT IV:**

**Topological spaces (Continued):** Open bases and open subbases–Weak topologies–The function algebras C (X, R) and C(X,). (Chapter 3: Sections 18 to 20 of the prescribed text book).

**Compactness:** Compact spaces- Heine-Borel theorem. (Chapter 4: Section 21). 12 Hours

**UNIT V:**

**Compactness (Continued):**  Product of Spaces – Tychonoff’s theorem and locally Compact spaces – Compactness for metric spaces – Ascoli theorem. (Chapter 4: Sections 22 to 25 of the prescribed text book). 12 Hours

**Prescribed book:**

Introduction to Topology by G. F. Simmons, Mc. Graw-Hill book company.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**I SEMESTER MATHEMATICS**

**M 104 DIFFERENTIAL EQUATIONS MAX.MARKS:100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To familiarize with essential concepts of real function theory that help to grasp the theory of ordinary differential equations.

CO 2: To introduce basic theorems in theory of ordinary differential equations pertaining to existence, uniqueness, continuation of solutions.

CO 3: To understand dependence of solutions on initial conditions and parameters.

CO 4: To transform nth order differential equations in to differential systems and extend the theory to differential systems.

CO 5: To study the qualitative behaviour of solutions of homogeneous and nonhomogeneous linear equations and systems.

**UNIT I:**

Essential concepts from Real Function Theory – The basic problem -The fundamental

existence and uniqueness theorem –examples to demonstrate the theory- continuation of

solutions.

12 Hours

(Sections 10.1, 10.2 of the prescribed text book)

**UNIT II:**

Dependence of solutions on initial conditions – dependence of solutions on parameters

(causal function f) - Existence and Uniqueness theorems for systems – existence and

uniqueness theorems for Higher order equations – examples.

12 Hours

(Sections 10.3, 10.4 of the prescribed text book)

**UNIT III:**

Introduction to the theory of Linear differential systems – Theory and properties of

Homogeneous linear systems.

12 Hours

(Sections 11.1 - 11.3 of the prescribed text book)

**UNIT IV:**

Theory of non-homogeneous linear systems – Theory and properties of the nth order

homogeneous linear differential equations.

12 Hours

(Sections 11.4 - 11.6 of the prescribed text book)

**UNIT V:**

Theory of nth order Non-homogeneous Linear equations – Sturm theory – Sturm Liouville Boundary value problems.

12 Hours

(Sections 11.7, 11.8, 12.1 of the prescribed text book)

**Prescribed Text Book:**

Shepley L. Ross (2007). Differential Equations (3rd edition), Wiley India.

**Reference book:**

George F. Simmons (2017). Differential Equations with Applications and Historical Notes (3rd edition). CRC Press. Taylor & Francis.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**I SEMESTER MATHEMATICS**

**M 105 LINEAR ALGEBRA MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To introduce the essential concepts of linear transformations on finite dimensional vector spaces.

CO 2: To understand the utilization of ordered basis to represent linear transformations by matrices.

CO 3: To select a single linear operator on finite dimensional vector space and to take it apart to see what makes it tick.

CO 4: To characterise the smallest subspace of a vector space which is invariant under linear operator.

CO 5: To decompose a linear operator on a finite dimensional vector space into a direct sum of operators which are elementary.

**Unit I**:

Introduction, Characteristic Values, Similar Matrices, Diagonalizable Operators, Annihilating

Polynomials, Minimal Polynomials, Cayley – Hamilton Theorem.

12 Hours

**(**Sections 6.1 - 6.3 of Chapter 6 in the Prescribed Text Book)

**UNIT II:**

Invariant Subspaces, T-conductor of a vector, T-annihilator of a vector, Simultaneous

Triangulation; Simultaneous Diagonalization.

12 Hours

**(**Sections 6.4 - 6.5 of Chapter 6 in the Prescribed Text Book)

**Unit III:**

Direct-Sum Decompositions, Projections, Invariant Direct Sums, The Primary Decomposition Theorem.

12 Hours

(Sections 6.6 – 6.8 of Chapter 6 in the Prescribed Text Book)

**Unit IV:**

Cyclic Subspaces and Annihilators, T-cyclic Subspace Generated by a Vector, Companion

Matrices, Complementary Subspaces, I-admissible Subspaces, Cyclic Decompositions and

Rational form, Generalized Cayley – Hamilton Theorem Invariant Factors.

12 Hours

(Sections 7.1, 7.2 of Chapter 7 in the Prescribed Text Book).

**Unit V:**

The Jordan Forms, Elementary Jordan Matrix with Characteristic Value, Computation of

Invariant Factors, Elementary Matrices, Smith Normal Forms, Summary; Semi-Simple

Operators.

12 Hours

(Sections 7.3 – 7.5 in the Prescribed Text Book)

**Prescribed Book:**

Linear Algebra by Kenneth Hoffman and Ray Kunze, prentice-Hall India Pvt. Ltd, 2nd Edition, New Delhi.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**II SEMESTER MATHEMATICS**

**M 201 ALGEBRA–II MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To understand the concept of extensions of a field, based on the study of irreducible polynomials.

CO 2: To understand the concept of normal extensions and separable extensions based on the study multiplicity of roots of a polynomial.

CO 3: To introduce the concept of group of automorphisms on a field. To introduce fixed fields. To prove the fundamental theorem of Galois theory.

CO 4: To apply Galois theory and prove the fundamental theorem of algebra. To study the properties of nth cyclotomic polynomial.

CO 5: To understand Galois theory and study its applications.

**UNIT I:**

**Algebraic extension of fields:** Irreducible polynomials and Eisenstein’s criterion Adjunction of roots-Algebraic Extensions-Algebraically closed fields.

12 Hours

(Sections 15.1 to 15.4 of the Chapter 15 in the prescribed text book.)

**UNIT II:**

**Normal and separable extensions:** Splitting fields- Normal extensions-multiple roots-finite fields.

12 Hours

(Sections 16.1 to 16.4 of the Chapter 16 in the prescribed text book.)

**UNIT III:**

**Normal and separable extensions:** separable extensions

**Galois Theory:** Automorphism groups and fixed fields- fundamental theorem of Galois Theory.

12 Hours

(Section 16.5 of the Chapter 16 and Sections 17.1 to 17.2 of the Chapter 17 in the prescribed text book.)

**UNIT IV:**

**Galois Theory:** Fundamental theorem of algebra.

**Galois Theory and Applications of Galois Theory to classical problems:** Roots of unity and cyclotomic polynomials-cyclic extensions-polynomials solvable by radicals- symmetric functions.

12 Hours

(Section 17.3 of the Chapter 17 and sections 18.1and 18.2 of the Chapter 18 in the prescribed text book.)

**UNIT V:**

**Galois Theory and Applications of Galois Theory to classical problems:** Polynomials solvable by radicals- symmetric functions-Ruler and compass constructions.

12 Hours

(Sections 18.3 and 18.4 of the Chapter 18 in the prescribed text book.)

**Prescribed Text Book:**

Basic Abstract Algebra: P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Second edition, Cambridge University Press, printed and bound in India at Replika Press Pvt. Ltd., 2001.

**ReferenceBooks:**

1. Topics in Algebra: I. N. Herstein, 2nd Edition, John Wiley & Sons

2. Algebra: Serge Lang, Revised Third Edition, Springer

3. Algebra: Thomas W. Hungerford, Springer

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**II SEMESTER MATHEMATICS**

**M 202 REAL ANALYSIS–II MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: Discuss the most important aspects of the problems that arise when limit processes are interchanged.

CO 2: Study the approximation of continuous complex function and its generalization and an introduction of power series.

CO 3: Study of exponential and logarithmic functions, the trigonometric functions and Fourier series and their properties.

CO 4: Discuss linear transformations on finite-dimensional vector spaces over any field of scalars and derivative of functions of several variables.

CO 5: Study the method of solving implicit functions. Interesting illustration of the general principle that the local behaviour of a continuously differentiable mapping near a point. Further study of derivatives of higher order and differentiation of integrals.

**UNIT I:**

**Sequences and Series of the Functions:** Discussion on the Main Problem, Uniform Convergence, Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation.

12 Hours

Chapter 7, Section 7.1 to 7.18, of the Text Book.

**UNIT II:**

**Sequences and Series of the Functions (continued):** The Stone-Weierstrass Theorem

Power Series

12 Hours

Chapter 8, Sections 8.1 to 8.5, of the Text Book.

**UNIT III:**

**Some Special Functions:** The Exponential and Logarithmic Functions, The Trignometric

functions, Fourier Series, Parseval’s theorem.

12 Hours

Chapter 8, Sections 8.6 and 8.7, 8.9 to 8.16, of the Text Book.

**UNIT IV:**

**Functions of Several Variables:** Linear Transformations, Differentiation, The Contraction Principle, The Inverse Function theorem. The implicit function theorem.

12 Hours

Chapter 9, Sections 9.4 to 9.25, of the Text Book.

**UNIT V:**

**Functions of several variables (continued):** The implicit Function theorem, The Rank theorem, Determinants, Derivatives of higher order, Differentiation of integrals.

12 Hours

Chapter 9, Sections 9.4 to 9.25, of the Text Book.

**Prescribed Text Book:**

Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3rd Edition, 1985.

**REFERENCE BOOK:**

Mathematical Analysis by Tom M. Apostol, Narosa Publishing House, 2nd Edition, 1985.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**II SEMESTER MATHEMATICS**

**M 203 TOPOLOGY–II MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To study Separation properties of Topological spaces, Urysohn’s lemma, Tietze’s extension theorem.

CO 2: To understand the concept of metrizability of a topological space, Urysohn’s imbedding theorem and one-point compactification of a topological space.

CO 3: To understand the concept of connected spaces, locally connected spaces, and totally disconnected spaces and their properties.

CO 4: To Prove Weirstrass approximation theorem and Stone - Weirstrass theorems.

CO 5: To study locally compact spaces and generalise Stone - Weirstrass theorems.

**UNIT I:**

Separation: spaces and Hausdorff spaces – Completely regular spaces and normal spaces – Urysohn’s lemma and the Tietze’s extension theorem. (Chapter 5: Sections 26 to 28 Prescribed text book).

12 Hours

**UNIT II:**

Separation (continued): The Urysohn imbedding theorem – The Stone – Chech compactification. (Chapter 5: Sections 29 to 30 Prescribed text book).

Connectedness: Connected spaces– connectedness of and . (Chapter 6: Section 31 Prescribed text book).

12 Hours

**UNIT III:**

Connectednedness (continued): The components of a space – Totally disconnected spaces –Locally connected spaces. (Chapter 6: Sections 32 to 34 Prescribed text book)

12 Hours

**UNIT IV:**

Approximation: The Weierstrass approximation theorem - The Stone-Weierstrass theorems. (Chapter 7: Section 35 to 36 Prescribed text book).

12 Hours

**UNIT V:**

Approximation (continued): Locally compact Hausdorff spaces – The extended Stone-Weierstrass theorems. (Chapter 7: Sections 37 to 38 Prescribed text book).

12 Hours

**Prescribed book:**

Introduction to Topology by G. F. Simmons, Mc.Graw-Hill book company.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**II SEMESTER MATHEMATICS**

**M 204 COMPLEX ANALYSIS MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To learn basic properties of power series and utilise this knowledge to construct analytic functions. To understand the relation between the Cauchy - Riemann equations and analytic functions. Study the nature and properties of Mobius transformation.

CO 2: To know about Power series expansion of analytic functions, significant properties analytic functions, zeros of analytic functions - gain knowledge pertaining to Liouville theorem, fundamental theorem of algebra, maximum modulus theorem and to know about index of a closed curve.

CO 3: To understand the three versions of Cauchy integral formula, Cauchy's theorem and Study Morera's theorem and its significance.

CO 4: Be aware of some applications of Cauchy theorem to count zeros of an analytic function and the open mapping theorem as a property of analytic function.

CO 5: Recognise and classify singularities of an analytic function - learn about residue theorem.

CO 6: Be aware of three versions of maximum modulus theorem and also the Swartz's lemma.

**UNIT I:**

Power series- Analytic functions- Analytic functions as mappings, Mobius transformations.

12 Hours

($1, $2,$3 of chapter-III of the prescribed text book)

**UNIT II:**

Power series representation of analytic functions- zeros of an analytic function -The index of a closed curve.

12 Hours

($2, $3, $4 of chapter-IV of the prescribed text book)

**UNIT III:**

Cauchy’s theorem and integral formula - Counting zeros; the open mapping theorem.

12 Hours

($5, $7 of chapter-IV of the prescribed text book)

**UNIT IV:**

Classifications of singularities- Residues and related results.

12 Hours

($1, $2 of chapter-V of the prescribed text book)

**UNIT V:**

The maximum principle – Schwarz’s lemma and related results.

12 Hours

($1, $2 of chapter-VI of the prescribed text book)

**Prescribed text book:**

Functions of one complex variable by J. B. Conway: Second edition, Springer International student Edition, Narosa Publishing House, New Delhi.

**ST. JOSEPH’S COLLEGE FOR WOMEN, VISAKHAPATNAM**

**II SEMESTER MATHEMATICS**

**M 205 DISCRETE MATHEMATICS MAX. MARKS: 100**

**w.e.f. 2021-22 SYLLABUS CREDITS: 4**

**COURSE OUTCOMES:**

After studying this course, students should be able to:

CO 1: To understand The Four Colour Theorem and applications in chemistry and physics.

CO 2: To familiarize the basic concepts of graphs and different types of graphs.

CO 3: To learn the modelling of Konigsberg Bridge Problem and Hamilton’s Game by graphs.

CO 4: To characterize graphs which are both Eulerain and Hamiltonian.

CO 5: To understand specific difference between modular and distributive lattices.

CO 6: To learn the importance of diamond and pentagon lattices.

**UNIT I:**

Basic Ideas, History, Initial Concepts, Summary, Connectivity, Elementary Results, Structure Based on Connectivity.

12 Hours

(Chapters – 1 & 2 of Text Book 1)

**Unit II:**

Trees, Characterizations, Theorems on Trees, Tree Distances, Binary trees, Tree Enumeration, Spanning trees, Fundamental Cycles, Summary.

12 Hours

(Chapter – 3 of Text Book 1)

**Unit III:**

Traversability, Introduction, Eulerian Graphs, Hamiltonian Graphs, Minimal Spanning Trees, J. B. Kruskal’s Algorithm, R. C. Prim’s Algorithm**.** (Chapter 4 of Text Book 1 and Section 7.5 of Text Book 2)

12 Hours

**Unit IV:**

Poset Definition, Properties of Posets, Lattice Definition, Properties of Lattices.

12 Hours

(Chapter 1-A of Text Book 3)

**Unit V:**

Definitions of Modular and Distributive Lattices and its Properties.

12 Hours

(Chapter 1-B of Text Book 3)

**Prescribed Text Book:**

1. Graph Theory Applications by L. R. Foulds, Narosa Publishing House, New Delhi.

2. Discrete Mathematical Structures by Kolman and Busby and Sharen Ross, Prentice Hall of India – 2000, 3rd Edition

3. Applied Abstract Algebra by Rudolf Lidl and Gunter Pilz, Published by Springer- Verlag.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**III Semester**

**M 301 Functional Analysis (Compulsory)**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To understand the concept of Banach spaces and Hilbert spaces.
2. Explain the concept of Projection on Hilbert and Banach spaces.
3. To study in detail the concept of determinants and the spectrum of an operator.

**Outcomes:**

After studying this course, students should be able to:

1. Explain the concept of normed, Banach & Hilbert spaces with standard examples and relation between them.
2. Demonstrate the continuous linear transformations and the Hahn-Banach theorem
3. Describe uniform boundedness theorem, open mapping theorem and closed graph theorem and their applications.
4. Summarize orthogonal vectors in a Hilbert space, orthogonal complement of a subset of a Hilbert space, orthonormal set and a complete orthonormal set.
5. Classify different operators on Hilbert space, which commute with their adjoints.
6. Discuss Spectral theorem and its applications.

**Syllabus:**

**UNIT-I**

Banach spaces: The definition and some examples, continuous linear transformation, The Hahn Banach theorem, the natural imbedding of N in N\*\*, The open mapping theorem.

Sections 46-50, Chapter 9.

**UNIT-II**

The conjugate of an operator, Hilbert spaces: The definition and some simple properties, orthogonal complements, orthonormal sets.

Section 51, Chapter 9 and Sections 52-54, Chapter 10.

**UNIT-III**

(Self) Adjoint, Normal, Unitary Operators: The conjugate space H\*, the adjoint of an operator, Self-adjoint operators, Normal and Unitary operators, Projections.

Sections 55-59, Chapter 10.

**UNIT-IV**

Finite-dimensional spectral theory: Matrices, determinants and the spectrum of an operator, the spectral theorem. A survey of the situation.

Sections 60-63, Chapter 11.

**Text Book:**

Introduction to Topology and Modern Analysis by G. F. Simmons, McGraw Hill Book Company. Inc-International student edition.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**III-SEMESTER**

**M 302(1)-NUMBER THEORY- I**

**(Revised/w.e.f. 2018-19 admitted batch)**

**Objectives:**

To study about the various Arithmetical functions and proving properties.

To learn about important subgroups of Arithmetical functions and Dirichlet product on functions.

To know the definition and basic properties of congruences, Residue classes and complete residue systems.

Important results about Euler Summation formula and its applications, some elementary theorems about distribution of primes.

To understand the behavior of various functions for a large value, the Big-oh notation and Asymptotic equality of functions.

Learning about Characters of finite abelian groups and Dirichlet’s theorem for primes.

**Outcomes:**

After studying this course, students should be able to:

1. Describe the important of arithmetical functions Mobius, Euler Totient, Mangold and divisor functions.
2. Explain prime number theorem and its equivalent conditions theorems, Abel’s identity.
3. Ability to solve applications of Euler- Fermat theorem, Lagrange’s theorem and Polynomial congruences with prime power moduli.
4. Discuss about The Chinese remainder theorem and Applications of the Chinese remainder theorem.
5. Ability to solve properties related to above mentioned functions and sums involving Dirichlet characters.

**Syllabus:**

**UNIT-I**

ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION:

Introduction- The Mobius function function µ (n) – The Euler totient function ϕ (n)- A relation connecting ϕ and µ - A product formula for ϕ (n)- The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function Λ(n)- multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville’s function - The divisor functions.

Chapter-2: - Articles 2.1 to 2.14

AVERAGES OF ARITHMETICAL FUNCTIONS:

Introduction- The big oh notation. Asymptotic equality of functions- Euler’s summation formula- Some elementary asymptotic formulas-The average order of d(n)- The average order of the divisor functions - The average order of ϕ (n). The partial sums of a Dirichlet product- Applications to µ (n) and Λ(n)- Another identity for the partial sums of a Dirichlet product.

Chapter -3: - Articles 3.1 to 3.7

**UNIT-II**

The partial sums of a Dirichlet product- Applications to µ (n) and Λ(n)- Another identity for the partial sums of a Dirichlet product.

SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMBERS:

Introduction- Chebyshev’s functions and - Relations connecting and - Some equivalent forms of the prime number theorem-Inequalities for π(n) and Shapiro’s Tauberian theorem- Applications of Shapiro’s theorem- An asymptotic formula for the partial sums - The partial sums of the Mobius function – The partial sums of the Mobius function.

Chapter -3: - Articles 3.10 &3.11 and Chapter-4: - Articles 4.1 to 4.9

**UNIT-III**

CONGRUENCES:

Definition and basic properties of congruences- Residue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p. Lagrange’s theorem- Applications of Langrage’s theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem- Polynomial congruences with prime power moduli.

Chapter -5: - Articles 5.1 to 5.9

**UNIT-IV**

FINITE ABELIAN GROUPS AND THEIR CHARACTERS:

Characters of finite abelian groups- The character group- The orthogonality relations- for characters- Dirichlet characters- Sums involving Dirichlet characters-The nonvanishing of L (1, χ) for real nonprincipal χ .

DIRICHLET’S THEOREM ON PRIMES IN ARITHMETIC PROGRESSIONS:

Introduction- Dirichlet’s theorem for primes of the form 4n-1 and 4n+1- The plan of the proof of Dirichlet’s theorem- Proof of Lemma 7.4- Proof of Lemma 7.5- Proof of Lemma 7.6- Proof of Lemma 7.7- Proof of Lemma 7.8- Distribution of primes in arithmetic progressions.

Chapter 6: - Articles 6.5 to 6.10 and Chapter 7: - 7.1 to 7.9

**TEXT BOOK:**

Introduction to Analytic Number Theory- By T. M. APOSTOL-Springer Verlag-New York, Heidalberg-Berlin-1976.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**III-SEMESTER**

**M 303(1) LATTICE THEORY-I**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To understand the concept of partially ordered sets.
2. To study the concept of lattices, complete lattices, Distributive and Modular lattices.
3. Explain the concept of Valuation of lattice.

**Outcomes:**

After studying this course, students should be able to:

1. Describe about the minimum and maximum conditions.
2. Discuss about the Jordan-Dedekind chain condition.
3. Draw the diagrams of POSETS and LATTICES.
4. Summarizing the importance of irreducible and prime elements of lattice.
5. Characterize complete lattices and conditionally complete lattices.
6. Discuss Dedekind’s modularity criterion.
7. Summarize the valuation of lattice, Metric and Quasi metric lattice.

**Syllabus:**

**UNIT-I**

Partially Ordered sets- Diagrams- Special subsets of a poset –length- lower and upper bounds- the minimum and maximum condition- the Jordan Dedekind chain conditions – Dimension functions.

Sections 1 to 9 of chapter I of the prescribed text book

**UNIT-II**

Algebras-lattices- the lattice theoretic duality principle- semi lattices- lattices as posets-diagrams of lattices- semi lattices, ideals-bound elements of Lattices-atoms and dual atoms-complements, relative complements, semi complements-irreducible and prime elements of a lattice- the homomorphism of a lattice-axioms systems of lattices.

Sections 10 to 21 of chapter II of the prescribed text book

**UNIT-III**

Complete lattices- complete sub lattices of a complete lattice- conditionally complete lattices- lattices – compact elements, compactly generated lattices- sub algebra lattice of an algebra-closure operations- Galois connections, Dedekind cuts- partially ordered sets as topological spaces.

Sections 22 to 29 of chapter III of the prescribed text book

**UNIT-IV**

Distributive lattices-infinitely distributive and completely distributive lattices-modular lattices characterization of modular and distributive lattices by their sub lattices- distributive sublattices of modular lattices- the isomorphism theorem of modular lattices, covering conditions-meet representations in modular and distributive lattices- some special subclasses of the class of modular lattices-preliminary theorems – modular lattices of locally finite length- the valuation of a lattice, metric and quasi metric lattices- complemented modular lattices.

Sections 30 to 40 of Chapters IV and V of the prescribed text book

**Prescribed Text Book:**

Introduction to Lattice Theory, by Gabor Szasz, Academic Press, New York.

**Book for reference:**

General Lattice Theory by G. Gratzer, Academic Press, New York.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**III-SEMESTER**

**M 304(1) COMMUTATIVE ALGEBRA- I**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To study the theory classes and ideals in Commutative Rings.
2. To learn about operations on ideals, extension and contraction of ideals in commutative rings.
3. To Explain the concept of modules, operations on modules and tensor product of modules and algebras.
4. Describe the Important results concern tensor products and exact sequences of module, noetherian rings.
5. To understand the model Rings of fractions.
6. Learning about uniqueness on Primary decomposition of ideals.

**Outcomes:**

After studying this course, students should be able to:

1. A comprehensible clarifying of commutative rings and their properties.
2. A clear and brief illustration of proofs related to Commutative Rings, Modules and Ideals.
3. Describe tensor properties of modules, exact sequences and flat modules.
4. Have a clear model about localization of rings at a prime ideal.
5. Ability to solve the properties of extended and contracted ideals in ring of fractions.

**Syllabus:**

**UNIT-I**

Rings and ring homomorphism, ideals, quotient rings, zero divisors, Nilpotent elements, units, prime ideals and Maximal ideals, nil radical and Jacobson radical, operations on ideals, Extensions and contractions.

**UNIT-II**

Modules and module homomorphisms, Sub modules and quotient modules, operations on sub modules, Direct sum and product, finitely generated modules, exact sequences, Tensor product of modules, Restriction and extension of scalars, Exactness properties of the tensor product, algebras, tensor product of algebras.

**UNIT-III**

Local Properties, Extended and contracted ideals in rings of fractions.

**UNIT-IV**

Primary decompositions (Content and extent chapters 1 to 4 of the prescribed text book).

**Prescribed text book:**

Introduction to commutative algebra, By M. F. ATIYAH and I.G. MACDONALD, Addison-Wesley publishing Company, London.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**III semester**

**M 305: Calculus of Variations (Compulsory)**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. Fully understand the concept of functional and Euler’s equations.
2. Describe the branchistochrone problem mathematically and solve it.
3. Solve simple initial and boundary value problems by using several variable calculus.
4. Compute one sided variations, moving boundaries, reflection and reflection of extremals.
5. Formulate important results and Clairaut’s theorem and Noether’s theorem.
6. Be through with methods for solving Elastic bodies and Electro statics problems.

**Outcomes:**

After studying this course, students should be able to:

1. Calculus of variations is a field in mathematics that deals with finding shortest path between two points.
2. The calculus of variations is a powerful technique for the solution of problems in dynamics of rigid bodies, optimization of orbits and vibration problems.
3. Calculus of variations helps to formulate geodesic problems on a plane and sphere.
4. They range from the problem in geometry of finding the shape of a soap bubble, a surface that minimized its surface area.
5. Obtain the configuration of piece of elastic that minimizes its energy.
6. Analyse the problems in which one wishes to find the minima of extrema of some quantity over a system that has functional degrees of freedom.

**Syllabus:**

**Unit 1**

Variational Problems with Fixed Boundaries – The Concept of variation and its properties- Euler’s equation-Variational problems for functionals- functionals dependent on higher order derivatives- functionals dependent on functions of several independent variables-Some application problems of Mechanics

(Sections 1.1-1.5,1.7 of Chapter 1).

**Unit 2**

Variational Problems with Moving Boundaries – Functionals involving first order derivative-Variational problem with a moving boundary for a functional dependent on two functions-one sided variations-reflection and reflection of extremals.

(Sections 2.1-2.4 of Chapter 2).

**Unit 3**

Sufficient conditions for an extremum – second variation-poisson bracket-The Hamilton-Jacobi equation-Clairaut’s Theorem-Noether’s theorem

(Sections 3.5,3.8,3.10-3.12 of Chapter 3).

**Unit 4**

Variational problems – Various constraints – Isoperimetric problems – Problems of Mayer and Bolza-Equilibrium problems for Elastic Bodies – problem of Electro statics

(Sections 4.1 – 4.5 of Chapter 4).

**Prescribed Text book:**

A.S. Gupta, Calculus of Variations with Applications, PHI Learning Private Limited, 2009.

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**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**IV-SEMESTER**

**M 401 MEASURE THEORY (Compulsory)**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To introduce the concept of measure and integration with respect to a measure.
2. To study basic properties of measurable functions.
3. To Understand how lebesgue measure on R is defined.
4. To study the concept of Absolute continuity and convex functions.
5. To introduce the concept of Bounded linear functionals on the spaces.

**Outcomes:**

After studying this course, students should be able to:

1. Analyse measurable sets and lebesgue measure.
2. Construct lebesgue’s measure on the real line and in n-dimensional Euclidean space.
3. Discuss how measures may be used to construct integrals.
4. Distinguish the relation between differentiation and lebesgue integration.
5. Discuss littlewood’s three principles, Egoroff’s theorem, Fatou’s lemma and Monotone convergence theorem.

**Syllabus:**

**UNIT-I**

Lebesgue measure: Introduction, Outer measure, measurable sets and Lebesgue measure, A non-measurable set, measurable functions, Littlewood’s three principles.

Chapter 3 of the text book

**UNIT-II**

The Lebesgue Integral: The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, the integral of nonnegative function, the general Lebesgue integral, convergences in measure.

Chapter 4 of the text book

**UNIT-III**

Differentiation and integration: Differentiation of monotone functions, Functions of bounded variation and differentiation of an integral, Absolute continuity, and convex functions.

Chapter 5 of the text book

**UNIT-IV**

The classical Banach spaces: The Lp-spaces, The Minkoswki and Holder inequalities, convergence and completeness, approximation in , Bounded linear functionals on the spaces.

Chapter 6 of the text book

**TEXT BOOK:**

Real Analysis by H. L. Royden, Macmillan Publishing Co. Inc. 3rd Edition, New York, 1988.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**IV-SEMESTER**

**M 402(1) NUMBER THEORY- II**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To be taught about periodic arithmetical functions and Ramanujan’s sum and generalizations.
2. To learn the quadratic reciprocity law and Gauss Lemma.
3. To become skilled at Primitive roots, reduced residue systems, quadratic residues.
4. To deduct important theorems on Real-valued Dirichlet characters Primitive Dirichlet characters, Integral representation for the Hurwitz zeta function.
5. To understand the function defined by Dirichlet series, Multiplication of Dirichlet series, Euler Products, The half-plane of convergence of a Dirichlet series.

**Outcomes:**

After studying this course, students should be able to:

1. To prove a number of results on functions periodic modulo k and Quadratic residues- Legendre’s symbol and its properties.
2. Expertise on applications of the reciprocity law.
3. Able to prove the existence of primitive roots and p for odd primes p.
4. Discuss the analytic properties of Dirichlet series and the halfplane of absolute convergence of a Dirichlet series.
5. Ability to prove properties of the gamma function and Hurwitz’s formula for ζ(s, a).

**Syllabus:**

**UNIT-I**

PERIODIC ARITHMETIAL FUNCTIONS AND GAUSS SUMS:

Functions periodic modulo k- Existence of finite Fourier series for periodic arithmetical functions- Ramanujan’s sum and generalizations- Multiplicative properties of the sums Gauss sums associated with Dirichlet characters-Dirichlet characters with nonvanishing Gauss sums.

QUADRATIC RESIDUES AND THE QUADRATIC RECIPROCITY LAW:

Quadratic residues- Legendre’s symbol and its properties- Evaluation of (-1/p) and (2/p)- Gauss Lemma-The quadratic reciprocity law-Applications of the reciprocity law- The Jacobi symbol Applications to Diophantine equations- Gauss sums and the quadratic reciprocity law.

Chapter 8: - Articles 8.1 to 8.6 and Chapter 9: - Articles 9.1 to 9.9

**UNIT-II**

PRIMITIVE ROOTS:

The exponent of a number mod m. Primitive roots- Primitive roots and reduced residue systems-The nonexistence of primitive roots mod for α ≥ 3- The existence of primitive roots and p for odd primes p. Primitive roots and quadratic residues- The existence of primitive roots mod - The existence of primitive roots mod 2 - The nonexistence of primitive roots in the remaining cases- The number of primitive roots mod m. The index calculus- Primitive roots and Dirichlet characters-Real-valued Dirichlet characters mod Primitive Dirichlet characters mod .

**UNIT-III**

DIRICHLET SERIES AND EULER PRODUCTS:

The half- plane of absolute convergence of a Dirichlet series, the function defined by Dirichlet series, Multiplication of Dirichlet series, Euler Products, The half-plane of convergence of a Dirichlet series, Analytic properties of Dirichlet series, Dirichlet series with non-negative coefficients.

Chapter- 11: - Articles 11.1 to 11.7.

**UNIT-IV**

Properties of the gamma function, Integral representation for the Hurwitz zeta function, A contour integral representation for the Hurwitz zeta function, The analytic continuation of the Hurwitz zeta function, Analytic continuation of ζ(s), L(s, χ) , Hurwitz’s formula for ζ(s, a), The functional equation for Riemann zeta function.

Chapter- 12: - Articles 12.1 to 12.8.

**TEXT BOOK:**

Introduction to Analytic Number Theory- By T. M. APOSTOL- Springer Verlag-New York, Heidalberg-Berlin-1976.

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**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**IV-SEMESTER**

**M 403(1) LATTICE THEORY-II**

**(Revised/w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To understand the concept of Boolean Algebras.
2. To study in detail semi modular lattices and ideals of lattices.
3. Describe Congruence relations.

**Outcomes:**

After studying this course, students should be able to:

1. Illustrate the class of Boolean algebras and Boolean rings are same.
2. Describe the concept of linear dependence.
3. Characterize prime ideals and maximal ideals.
4. Discuss Boolean algebra is a field of sets.
5. Discuss the necessary and sufficient condition that the lattice is distributive related with ideals is the kernel of at least one congruence relation

**Syllabus:**

**UNIT-I**

Boolean algebras, De Morgan formulae- Complete Boolean algebras- Boolean algebras and Boolean rings- The algebra of relations- The lattice of propositions- Valuations of Boolean algebras.

Sections 42 to 47 of chapters VI of the prescribed text book

**UNIT-II**

Birkhoff lattices- Semi modular lattices- Equivalence lattices- Linear dependence- Complemented semi modular lattices.

Sections 48 to 52 of chapters VII of the prescribed text book

**UNIT-III**

Ideals and dual ideals, Ideal chains- Ideal lattices- Distributive lattices and rings of sets.

Sections 53 to 55 of chapters VIII of the prescribed text book

**UNIT-IV**

Congruence relation of an algebra- Permutable equivalence relations- The Schreier refinement theorem in arbitrary algebras- Congruence relations of lattices- Minimal congruence relations of some subsets of a distributive lattice- The connection between ideals and congruence relations of a lattice.

Sections 56 to 61 of chapters IX of the prescribed text book

**Prescribed text book:**

Introduction to Lattice Theory by Gabor Szasz, Academic Press, New York.

**Books for reference:**

General Lattice Theory by G. Gratzer, Academic Press, New York.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**IV-SEMESTER**

**M 404(1) COMMUTATIVE ALGEBRA-II**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. To be taught the notion of Integral dependence.
2. To become skilled at the Chain Conditions on modules to establish strong theorems in commutative rings.
3. To deduct important theorems using Noetherian condition and proving Hilbert basis theorem.
4. To understand the importance of Artin rings and learning about a special kind of domains called Dedekind domains.

**Outcomes:**

After studying this course, students should be able to:

1. To prove a number of results on Integral dependence.
2. To learn Going-Up and Going-Down theorems concerning prime ideals in an integral extension.
3. To study valuation rings of a given field of fractions.
4. Discuss the basic results in the dimension theory for local rings.
5. Ability to establish important results on Noetherian domains of dimensions zero and Noetherian integral domains of dimension one.

**Syllabus:**

**UNIT-I**

Integral dependence, the going-up theorem-Integrally closed integral domains, the going down theorem, valuation rings.

**UNIT-II**

Chain Conditions

**UNIT-III**

Noetherian rings- Primary decomposition of Noetherian rings, Artin rings

**UNIT-IV**

Discrete valuation rings, Dedekind domains, Fractional ideals.

Content and extent of Chapters 5 to 9 of the prescribed text book.

**Prescribed Text Book:**

Introduction to commutative algebra by M. F. Atiya and I. G. Macdonald, Addison-Wesley Publishing Company, London.

**ST. JOSEPH’S COLLEGE FOR WOMEN**

**DEPARTMENT OF MATHEMATICS**

**M.Sc. MATHEMATICS**

**IV-SEMESTER**

**M 405 PARTIAL DIFFERENTIAL EQUATIONS (Compulsory)**

**(Revised w.e.f. 2018-19 admitted batch)**

**Objectives:**

1. Understand basic properties of standard PDE’s.

2. Apply the range of technique to find solutions of standard PDE’s.

3. Demonstrate accurate and efficient use of Fourier analysis techniques and their applications in the theory of PDE’s.

4. Demonstrate capacity to model physical phenomena using PDE’s and Cauchy’s problems.

5. Apply problem solving using concepts and techniques from PDE’s and Fourier analysis applied to diverse situations in physics, engineering, financial mathematics and in other mathematical contexts.

**Outcomes:**

After studying this course, students should be able to:

1. Discuss the fundamental axioms in mathematics and capability of developing ideas based on them.

2. Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialization.

3. Utilize techniques for solving mathematical models and their real implementation problems.

4. Nature problem solving skills, thinking creativity through assignments.

5. Expertise on a wide range of mathematical techniques and application of mathematical methods/tools in other scientific and engineering domains.

**Syllabus:**

**Unit-I**

First Order P.D.E.: Curves and Surfaces – Genesis of First Order P.D.E. – Classification of Integrals – Linear equations of the First Order - Pfaffian Differential Equations – Compatible Systems – Charpit’s Method- Jacobi’s Method.

(Chapter 1 – sections 1.1 – 1.8)

**Unit-II**

Second Order P.D.E.: Genesis of Second Order P.D.E. – Classification of Second Order P.D.E.,One Dimensional Wave equation: Vibrations of an Infinite String – Vibrations of Semi infinite String Vibrations of String of Finite Length – Riemann’s Method – Vibrations of a String of Finite Length (Method of Separation of Variables).

(Chapter 2 – section 2.1 – 2.3)

**Unit-III**

Laplace’s Equation: Boundary value Problems- Maximum and Minimum Principles- the Cauchy Problem-The Dirichlet Problem for the upper Half Plane- The Neumann Problem for the upper Half Plane- The Dirchilet Problem for a Circle- The Dirchilet Exterior Problem for a Circle- The Neumann Problem for a Circle- The Dirchilet Problem for a Rectangle

(Chapter 2 – section 2.4.1-2.4.9)

**Unit-IV**

Heat Conduction Problem: Heat Conduction – Infinite Rod Case - Heat Conduction –Finite Rod Case- Duhamel’s Principle: Wave Equation –Heat Conduction Equation. Integral Surfaces through a Given Curve- Quasi Linear Equations.

(Chapter 2 – sections 2.5, 2.6 and Chapter 1 –sections 1.9, 1.10)

**Text book:**

T. Amarnath, An Elementary Course in Partial differential equations, Second Edition, Narosa Publishing House.

**Reference Book:**

Ian Sneddon, Elements of Partial Differential Equations, McGraw-Hill International Editions.