ST.JOSEPH'S COLLEGE FOR WOMEN (AUTONOMOUS), VISAKHAPATNAM

VII SEMESTER **MATHEMATICS** TIME: 4Hrs/Week

M7303(3) **NUMBER THEORY** Max.Marks:100

w.e.f .20AH Batch **SYLLABUS**

**Course Objectives: To enable the students to**

- Introduction to divisibility and prime numbers, arithmetical functions, notation and asymptotic equality of functions, Chebyshev's functions and their relations
- Understanding the concept of greatest common divisor (GCD), Mobius function and Euler quotient function, Euler's summation formula and asymptotic formulas, Shapiro's Tauberian theorem and its applications
- Learning the Euclidean algorithm for finding GCD, Exploring Dirichlet multiplication and its properties, the prime number theorem
- Investigating the divisor functions and generalized convolutions, sums of Dirichlet products and their identities, polynomial congruences modulo p and Lagrange's theorem.

**Learning Outcomes:**

**After successful completion of the course, students will be able to**

- understand the fundamental theorem of arithmetic and problem solving skills
- understand Mobius function, Euler quotient function, The Mangoldt function , Liouville'sfunction, The divisor functions and the generalized convolutions.
- Understand Euler's summation formula, application to the distribution of lattice points andthe applications to $\mu(n)$ and $\Lambda(n)$
- Understand Shapiro's Tauberian theorem and its applications
- Understand residue classes and complete residue systems, application of Fermat's theorem and applications of Shapiro's theorem

## UNIT-I
## The Fundament Theorem of Arithmetic

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, Thegreatest common divisor of more than two numbers (Chapter 1 of the Text Book)

## UNIT-II
## Arithmetical Functions and Dirichlet Multiplication

Introduction- The Mobius function $\mu(n)$ – The Euler quotient function $\varphi(n)$ - A relation connecting $\varphi$ and $\mu$ - A product formula for $\varphi(n)$ - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function$\Lambda(n)$- Multiplicative functions- Multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function $\lambda(n)$ - The divisor functions $\sigma\alpha(n)$ - Generalized convolutions.(Chapter-2:- Articles 2.1 to 2.14)

## UNIT-III
## Averages of Arithmetical Functions

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summationformula- Some elementary asymptotic formulas-The average order of d(n)- The averageorder of the divisor functions $\sigma\alpha$ (n)- The average order of $\varphi$ (n)- An application to thedistribution of lattice points visible from the origin- The average order of $\mu$ (n) and $\Lambda$(n)-The partial sums of a Dirichlet product- Applications to $\mu$ (n) and $\Lambda$ (n)- Another identityfor the partial sums of a Dirichlet product.(Chapter -3:- Articles 3.1 to 3.12)

## UNIT-IV
## Some Elementary Theorems on The Distribution of Prime Numbers

Introduction- Chebyshev's functions $\psi$ (x) and $\upsilon$ (x) - Relations connecting $\upsilon$(x) and $\pi$(x) - Some equivalent forms of the prime number theorem-Inequalities for $\pi$ (n) and $P_n$ - Shapiro's Tauberian theorem- Applications of Shapiro's theorem- An asymptotic formulafor the partial sums $\Sigma p \leq x$ (1 / p) - The partial sums of the Mobius function. Chapter-4:- Articles 4.1 to 4.9

## UNIT-V
## Congruences

Definition and basic properties of congruences- Resudue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p. Lagrange's theorem- Applications of Lagrage's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applicationsof the Chinese remainder theorem- Polynomial congruences with prime power moduli.
Chapter -5:- Articles 5.1 to 5.9

**Activities:**

1. Assignments
2. Student Seminars and Guest Lecturers
3. Problem Solving Sessions

**Text Book:**

Introduction to Analytic Number Theory- By T.M.APOSTOL- Springer Verlag-New York,Heidalberg-Berlin-1976.

**References :**

**1.** G.A.Jones and J.M.Jones, Elementary Number Theory, Springer